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T i t l e : PREDICTION OF ARRIVAL TIME DEPENDENT DELAY
VARIABILITY AT SIGNALIZED INTERSECTIONS

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PREDICTION OF ARRIVAL TIME DEPENDENT DELAY VARIABILITY AT SIGNALIZED INTERSECTIONS

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ABSTRACT

This paper describes the derivation of two analytical models for predicting the mean and variance of delay that a vehicle will experience when traversing a fixed-time signalized intersection approach at a known future point of time. This delay, referred to as arrival time dependent delay in this paper, differs from the traditional average delay estimate used in intersection performance analysis in that it is a function of the time at which the vehicle transverse the link. Arrival time dependent estimates of the mean and variance of delay are important for the successful deployment of many Intelligent Transportation Systems such as in-vehicle route guidance systems and Advanced Traffic Management Systems. The models presented in this paper are developed on the basis of an analysis of delay under two extreme traffic conditions: highly undersaturated and highly oversaturated conditions. A discrete cycle-by-cycle simulation model is used to generate data for calibrating and validating the proposed models. The analysis indicates a remarkable agreement ($R^2 > 0.99$) between the proposed analytical models and the simulation results.

INTRODUCTION

The ability to accurately predict future link travel times in road traffic networks is a critical component for many Intelligent Transportation Systems (ITS) applications such as in-vehicle route guidance systems (RGS) and advanced traffic management systems (ATMS). While most existing systems are utilizing near real-time traffic information, systems such as in-vehicle route guidance systems would provide substantially larger benefits to drivers if they possessed the ability to estimate future travel times. As a result of random fluctuations in travel demands, interruptions caused by traffic controls, unpredictable occurrences of traffic incidents and changes in weather conditions, link travel times in an urban traffic environment are highly stochastic and time-dependant. It has been increasingly recognized that, for many ITS applications, estimates of the variability of travel times are as important as estimates of the expected average travel times (Rouphail, 1995; Fu, 1996). For example, having knowledge of the variability of travel times on individual links makes it possible to explicitly consider the reliability of alternative routes in identifying optimal routes. By considering the travel time variability in fleet vehicle routing and scheduling process, more reliable schedules may be generated resulting in improved quality of service (Fu and Teply, 1997).

This paper addresses the problem of quantifying the variability of travel times on signalized arterials with specific focus on predicting both the mean and the variance of delay that a vehicle will experience if traversing a signalized approach. This delay is referred to as *arrival time dependent delay* to distinguish it from the average delay concept traditionally used in performance analysis of signalized intersections. The latter considers the average consequence of a signal control to a whole traffic flow during a given evaluation period while the former focuses on the delay experienced by vehicles arriving at a specific future time.

The problems of estimating delays at signalized intersections have been extensively studied in the literature, however the vast majority of the work has focused on developing performance models for predicting average delays. Detailed discussions of these average

delay prediction models have been provided by Allsop (1972), Newell (1982) and Hurdle (1984). However, much less work has been done to quantify the variability of delay at a signalized approach. Teply and Evans (1989) analyzed the delay distribution at a signalized approach for evaluating signal progression quality. They observed that most of the delay distributions are bimodal and a point estimator is not adequate to describe these distributions. By considering the cyclic overflow delay over time as a Markov chain, Kimber and Hollis (1979), Cronje (1983) and Olszewski (1993, 1994) developed numerical methods to calculate the average delay and time-dependant distribution of average cyclic delay. This type of model, while capable of completely specifying the delay distribution, requires substantial computational resources for calculating and storing state and transition probabilities and therefore is not well suited for use in real-time operating environments where future delays for a large number of links need to be quickly estimated.

The real-time prediction of travel time, in which delay is commonly a critical component, has recently received considerable attention. Previous research has focused primarily on developing statistical models for estimating average travel times on the basis of data from various sources, such as a historical database and real-time data from loop detectors and probe vehicles (Boyce et al., 1993; Abours, 1986; Takaba, 1991; Van Aerde et al., 1993). Despite these efforts, a satisfactory model for predicting arterial link travel time in real-time has not yet been developed (Sen et al., 1997a). Sen et al (1997b) observed that the probe-reported travel times are not statistically independent and the variance of the mean of travel times obtained from probe vehicles travelling on signalized arterial links does not approach zero with increasing sample size. These findings indicate that the sole use of travel time data from probe vehicles may never provide accurate estimates of arterial link travel times. As an alternative to using probe vehicle data, Roupail (1995) developed a model for estimating the distribution of delay that explicitly considers signal settings. However, this model does not consider delays caused by random overflow and therefore may underestimate the mean and variance of delay, especially under saturated traffic conditions.

This paper presents two approximate models for predicting the mean and variance of arrival time dependent delay. Section 2 outlines the methodologies applied to develop the approximate models. Section 3 presents the development of the approximate models. Section 4 describes the discrete cycle-by-cycle simulation model that was developed for calibrating and validating the proposed models. This simulation model is used in Section 5 to generate data for calibrating and validating the proposed models under a variety of signal operating conditions. Finally, Section 6 presents conclusions and recommendations.

METHODOLOGY

The delay that a particular vehicle experiences when it travels through a signalized intersection approach depends on a number of factors including the arrival flow rate and distribution, signal timings and the time when the vehicle arrives at the approach. In a real application environment, many of these factors are random variables, which makes accurate prediction of this delay a very complicated process. As an initial research effort,

this paper considers the following idealized road traffic, signal control and prediction conditions:

- i) The intersection approach consists of a single through lane controlled by a fixed-time signal. The approach has unlimited space for queuing and has a constant saturation flow rate;
- ii) The headways of vehicle arrivals at the approach follow a shifted negative exponential distribution and no overflow queue is present at the time when a prediction is performed. The traffic stream consists only of passenger car units (pcu);
- iii) The vehicle arrival time prediction horizon is discretized with a resolution equal to the signal cycle length. It is assumed that the cycle during which vehicles are expected to arrive at the approach link, but not the exact arrival time within the cycle, can be predicted. This assumption is justified as in practice it is likely not possible to predict the arrival time of a vehicle with greater accuracy. Furthermore, as cycle times are commonly less than or equal to 120 seconds, a resolution equal to the cycle time is likely finer than that expected in most practical applications (e.g. five minutes).

The proposed methodology is similar to the one applied to develop the traditional performance models for estimating average delay at a signalized intersection (Webster, 1958; Kimber and Hollis, 1979; Akcelik, 1981; Teply et al., 1995; Rouphail and Akcelik, 1990; Brilon and Wu 1990). As illustrated in Figure 1, the arrival time dependent delay at time t , noted as D_t , is considered to include two random components: arrival time dependent uniform delay and arrival time dependent overflow delay: (Equation 1)

$$D_t = D_{t1} + D_{t2} \quad (1)$$

where the uniform delay component, D_{t1} , is defined as the portion of delay that would be incurred by a vehicle when the approach is undersaturated and vehicle arrival times are uniformly distributed within the time interval t to t plus cycle length c_y $[t, t + c_y]$. The overflow delay component, D_{t2} , represents the portion of delay that is caused by temporary overflow queues resulting from the random nature of arrivals and by continuous overflow when the arrival rate during the time period $[0, t]$ exceeds the capacity. It is important to recognize that, with the assumption of constant saturation flow rate or capacity, the arrival time dependent delay at time t depends only on the cumulative number of arrivals at time t (N_t). *This means that* the realization of vehicle arrivals, or how N_t is accumulated, is irrelevant to the determination of the arrival time dependent delay, as shown in Figure 1. This property implies that the estimation of arrival time dependent delay for a time varying traffic demand is just the same as for a constant traffic demand as long as the average arrival rate is known.

Models for the mean and variance of arrival time dependent delay are developed separately for each of these two delay components. The approach taken to develop the model for mean arrival time dependent delay is similar to the one applied in the development of traditional average delay models. The mean uniform delay is estimated by assuming arrivals with uniform headways. The mean overflow delay is obtained

through the well-known coordination transformation technique based on models derived from steady-state stochastic queuing theory and deterministic queuing theory.

The variance of the arrival time dependent uniform delay is obtained from deterministic queuing theory while the variance of the overflow delay component is directly calibrated from simulation data. The functional form of the variance model for overflow delays is constructed on the basis of an analysis of the variance models under two traffic extremes: highly undersaturated and highly oversaturated conditions.

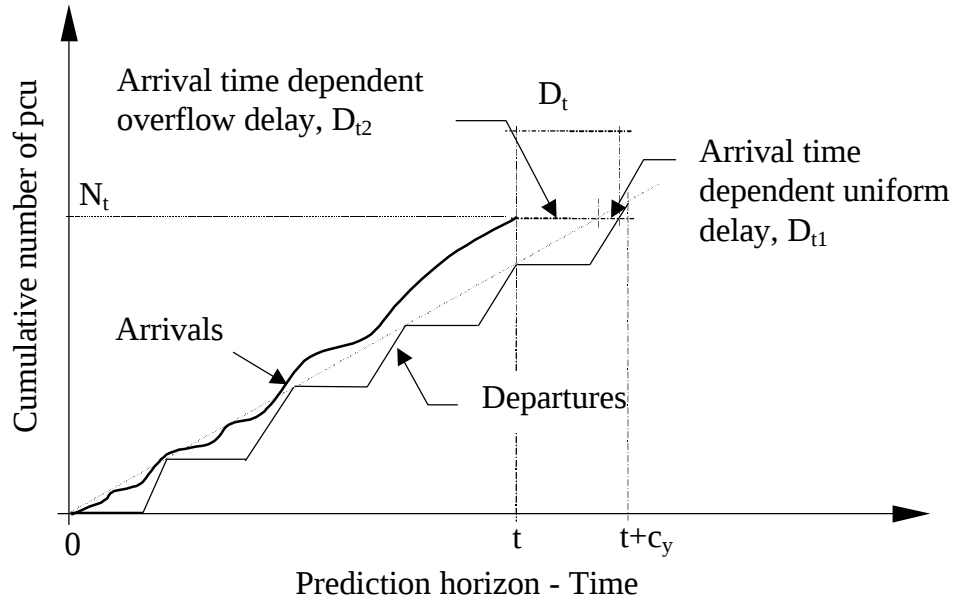


Figure 1. Queuing diagram illustrating the components of arrival time dependent delays

In order to verify and calibrate the proposed models, a simulation system was developed to generate data for a variety of operating conditions. The simulation model is first validated against several existing models and then used for calibrating and evaluating the proposed models.

APPROXIMATE MODELS FOR THE MEAN AND VARIANCE OF ARRIVAL TIME DEPENDENT DELAY

Mean of Arrival Time Dependent Delay

From Equation (1), the mean arrival time dependent delay can be expressed as sum of the means of arrival time dependent uniform delay and arrival time dependent overflow delay (Equation 2).

$$E[D_t] = E[D_{t1}] + E[D_{t2}] \quad (2)$$

The arrival time dependent uniform delay can be estimated by assuming that the vehicle arrivals are uniformly distributed with an average arrival rate of q_t . The traditional

uniform delay model for predicting the average uniform delay can therefore be used (Akcelik, 1981; Teply *et al.*, 1995):

$$E[D_1(t)] = \frac{c_y \cdot (1 - l)^2}{2(1 - l \cdot x_t)} \quad (3)$$

Where

- $E[D_{tl}]$ = mean arrival time dependent uniform delay (seconds)
- c_y = cycle time (seconds)
- λ = g_e/c_y
- g_e = effective green interval duration (seconds)
- q_t = average arrival flow rate from time 0 to time t (pcu/seconds).
- c_a = capacity (pcu/seconds), determined by $s l$, where s is the saturation flow rate (pcu/seconds)
- x_t = degree of saturation, defined as q_t/c_a
- x_l = minimum of $(1.0, x_t)$

The estimation of the arrival time dependent overflow delay component in Equation (2) is much more complicated as a result of the complex time-dependent stochastic nature of the queuing process, and currently there is no theory available for use to develop a single analytical model suitable across all saturation levels. Consequently, the well-known coordinate transformation technique, which has been successfully applied to develop traditional time-dependent delay models (i.e. models that consider the evaluation time period, not the time at which the vehicle arrives at the approach), is also used within this paper. The model for predicting this delay is established through coordinate transformation based on the steady-state model and the deterministic model for arrival time dependent overflow delay (Kimber and Hollis, 1979).

Arrival time dependent overflow delay for highly undersaturated traffic conditions can be analyzed with steady-state stochastic queuing theory by assuming that a steady-state can be reached at time t . The mean arrival time dependent overflow delay is therefore time-independent and its functional form can be established using the well-known coordination transformation method (Kimber and Hollis, 1979; Akcelik, 1988). However, instead of using the mathematical transformation process, the equation for the mean arrival time dependent overflow delay can be directly obtained from the equation for the average overflow delay described below.

With the deterministic queuing model, arrival time dependent overflow delay at time t is a linear function of the degree of saturation (Equation 4).

$$E[D_{t2}] = t(x_t - 1) \quad (4)$$

It can be observed that, for a given time t and degree of saturation x_t , the overflow delay predicted by Equation 4 is twice as large as the average overflow delay obtained from a deterministic model ($= [t(x_t - 1)]/2$, where t should be interpreted as the evaluation time, see e.g. Hurdle, 1982). With this relationship and the steady-state model, a model for the mean arrival time dependent overflow delay can be directly obtained from a traditional performance model by setting the evaluation time equal to $2t$, instead of applying the

coordination transformation process. For example, if the equation for average overflow delay from the Canadian Capacity Guide (Teply et al., 1984 and 1995) is used, then the mean arrival time dependent overflow delay at time t can be estimated by Equation (5)

$$E[D_2(t)] = 0.5t \left[(x_t - 1) + \sqrt{(x_t - 1)^2 + \frac{2x_t}{c_a \cdot t}} \right] \quad (5)$$

where t represents the point in time (in *seconds*) for which arrival time dependent overflow delay is to be computed. Note that the model predicts a zero overflow delay at time 0 (or as t approaches 0), which is reasonable because no queue is assumed to be present at $t=0$.

The same approach can be applied to average flow models other than the one used in this paper to obtain associated equations. Figure 2 schematically illustrates the transition curve represented by Equation (5) based on the steady-state model and deterministic model.

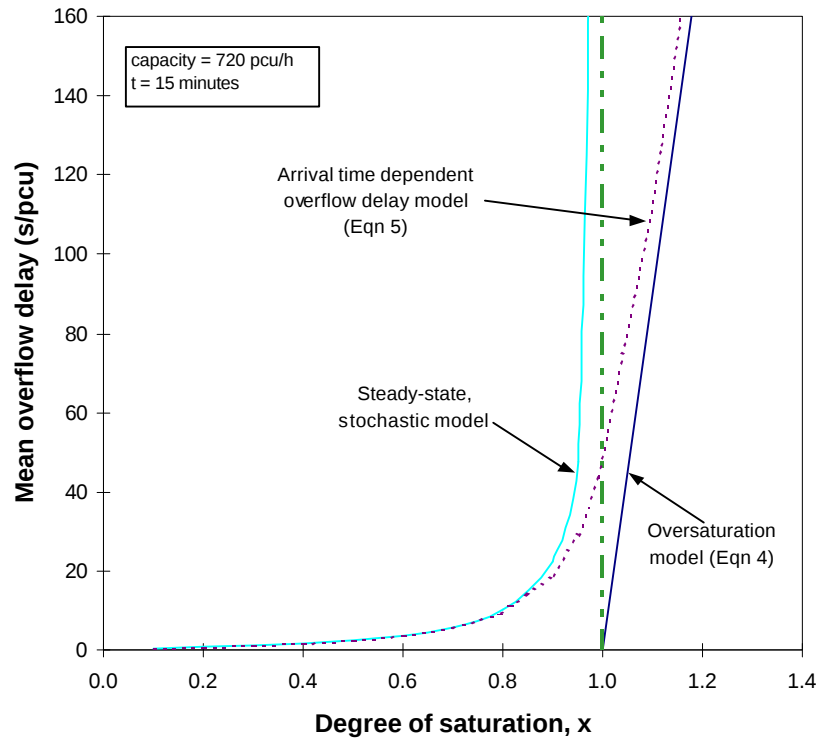


Figure 2. Models for the mean arrival time dependent overflow delay

It should be pointed out that the overflow model represented by Equation (5) is the result of mathematical manipulation with limited theoretical basis. In order to verify the validity of the model, a simulation analysis was performed and the results are presented in Section 4.

Having developed expressions for both the arrival time dependent uniform delay and the arrival time dependent overflow delay, Equation (3), representing the arrival time dependent total delay, can be rewritten as Equation (6).

$$E[D(t)] = \frac{c_y \cdot (1-l)^2}{2(1-l \cdot x_1)} + 0.5t \left[(x_t - 1) + \sqrt{(x_t - 1)^2 + \frac{2x_t}{c_a \cdot t}} \right] \quad (6)$$

Variance of Arrival Time Dependent Delay

If it is assumed that the arrival time dependent uniform delay and arrival time dependent overflow delay in Equation (1) are independent, the variance of total arrival time dependent delay is the sum of the variance of the uniform and overflow delays (Equation 7)

$$\text{Var}[D_t] = \text{Var}[D_{t1}] + \text{Var}[D_{t2}] \quad (7)$$

The variance of arrival time dependent uniform delay, $\text{Var}[D_{t1}]$, represents the variation of delay that would be experienced by vehicles arriving during the time interval $[t, t+c_y]$. This variation results from the uncertainty of the vehicle's arrival time during the cycle. The vehicle can arrive at any moment within the interval $[t, t+c_y]$ and thus experience variable delays as a result of the signal control. Van Aerde et. al. (1993) and Rouphail (1995) have developed an estimate of the variance of delay on the basis of a deterministic queuing model with vehicles arriving uniformly during the cycle. Rouphail's model, presented in Equation (8), is adopted in this paper to estimate the variance of arrival time dependent uniform delay.

$$\text{Var}[D_{t1}] = \frac{c_y^2 \cdot (1-l)^3 \cdot (1+3l-4lx_1)}{12(1-lx_1)^2} \quad (8)$$

In order to establish a model for the variance of arrival time dependent delay caused by overflow queue, two extreme traffic conditions are first investigated: undersaturated conditions ($x_t < 1.0$) and oversaturated conditions ($x_t > 1.0$). For undersaturated conditions, overflow delay experienced by a vehicle arriving during the time interval $[t, t+c]$ is mainly caused by occasional overflows of traffic from the previous cycle. The relationship between the variance of this delay and the degree of saturation can be obtained from the well-known Pollaczek-Khintchine formula for a M/G/1 system (for derivation, see e.g. Medhi, 1991):

$$\text{Var}[D_{t2}] = \frac{x_t \cdot (4 - x_t)}{12c_a^2 \cdot (1 - x_t)^2} \quad (9)$$

It should be emphasized that the above model is merely an approximate estimate of the variance even if a steady-state could be reached at time t because the actual departure at the signalized approach has a pulse service time. Nevertheless, the equation can be used to illustrate the qualitative relationship between the variance of arrival time dependent delay and the degree of saturation. With this assumption, the variance is time-independent and an infinite variance at time t would be predicted as the degree of saturation (x_t) approaches unity. In reality, at high degrees of saturation, the system is not likely to settle into a steady-state by time t . Consequently, it can be expected that

Equation (9) provides a reasonable approximation of the variance only under light traffic conditions.

If the intersection approach is highly oversaturated during the time period $[0, t]$ and there is always an overflow queue present during the period from time 0 to time t , the overflow queue at time t , Q_t , can be determined as the total arrivals minus the total departures (Equation 10).

$$Q_t = N_t - c_a \cdot t \quad (10)$$

The number of arrivals, N_t , is a random variable with a mean equal to $q_t \cdot t$. The delay experienced by a vehicle arriving at time t can then be simply determined on the basis of the overflow queue as expressed in Equation 11.

$$D_{t2} = \frac{N_t - c_a \cdot t}{c_a} \quad (11)$$

Since N_t is a random variable, the delay D_{t2} is also a random variable with variance determined by Equation (12):

$$\text{Var}[D_{t2}] = \frac{\text{Var}[N_t]}{c_a^2} \quad (12)$$

If the vehicle arrivals are Poisson distributed, the variance of the total arrivals is equal to the mean of the total arrivals:

$$\text{Var}[N_t] = q_t \cdot t \quad (13)$$

and therefore Equation (12) can be further expressed as:

$$\text{Var}[D_{t2}] = \frac{q_t \cdot t}{c_a^2} = \frac{t \cdot x_t}{c_a} \quad (14)$$

It must be emphasized that Equation (14) is valid only when there is an overflow queue present during the period from time 0 to time t . In reality, however, it is possible that no overflow queue exists at time t and consequently no overflow delay is experienced. Consequently, it can be concluded that Equation (14) represents an upper bound estimate of the variance of arrival time dependent overflow delay. The actual variance would be lower than that predicted by Equation (14), but the prediction error should become smaller as the degree of saturation increases, and the associated likelihood of overflow queuing increases.

Figure 3 depicts the relationships between the variances of arrival time dependent overflow delay as functions of the degree of saturation represented by Equation (9) and (14). Both curves are only appropriate within certain flow domains: either highly undersaturated or highly oversaturated traffic conditions. Consequently, it is hypothesized that the true relationship between the variance and the degree of saturation follows the dashed curve in Figure 3. This curve exhibits the unique double bending pattern that makes it difficult to derive the functional relationship directly from Equation

(9) and (14) through the traditional coordinate transformation technique. Therefore, the non-linear function, expressed in Equation 15, is proposed to model the true variance:

$$Var[D_2(t)] = \frac{t \cdot x_t}{c_a} e^{-\left(\frac{x_0}{x_t}\right)^b} \quad (15)$$

The parameters x_0 and b determine the shape of the delay curve and their values need to be calibrated. It can be observed that the proposed function has two desired attributes. First, the function is asymptotic to the model for oversaturated condition (Equation 14). Second, similar to the undersaturated model (Equation 9), the function goes to zero as x_t approaches zero. However, while these characteristics are necessary, they do not of themselves demonstrate that the proposed function is realistic. Therefore, data from a simulation model were used to calibrate appropriate values for x_0 and b and to validate the calibrated model, as discussed in Section 4.

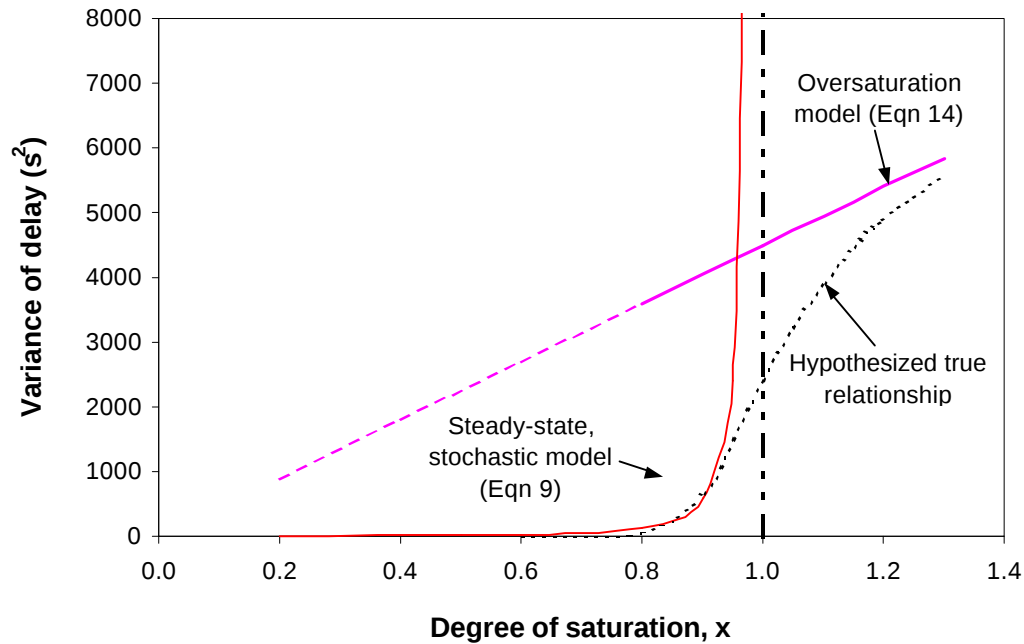


Figure 3. Models for the variance of arrival time dependent overflow delay

Having developed expressions for the variances of arrival time dependent uniform delay and overflow delay, the variance associated with the total arrival time dependent delay (Equation 7), can be expressed by Equation (16).

$$Var[D(t)] = \frac{c_y^2 \cdot (1-l)^3 \cdot (1+3l-4lx_1)}{12(1-lx_1)} + \frac{t \cdot x_t}{c_a} e^{-\left(\frac{x_0}{x_t}\right)^b} \quad (16)$$

SIMULATION ANALYSIS

In order to obtain data to calibrate and validate the proposed models, a discrete cycle-by-cycle simulation system was developed. The following sections briefly describe the design, verification and application of the simulation model.

Simulation Model

The simulation model explicitly models the delay that a vehicle experiences when traversing a signalized intersection approach. The approach is used exclusively for through traffic and controlled by a pre-timed traffic signal. The vehicle arrivals are randomly distributed with the vehicle headway following a shifted negative exponential distribution with a minimum headway equal to one second.

The vehicle discharge pattern during the green interval depends on the queue status at the approach. If there is no queue present when a vehicle arrives, then the vehicle can immediately be discharged without any delay. Otherwise, the vehicle must wait until the queued vehicles ahead of it discharge. The saturation flow rate is assumed to be 1800 pcu/h, which corresponds to a discharge headway of two seconds.

The simulation starts with no queue present and reset the queue size to zero whenever the elapsed clock time reaches a pre-specified time duration (80 minutes was used in this paper). The simulation terminates once the required total number of cycles has been simulated. The arrival time and delay associated with each vehicle are recorded for use in the analysis stage. Information such as the mean and variance of delays experienced by vehicles arriving during specific time intervals can then be derived.

Verification of the Simulation Model

Before the simulation model was used to generate data for calibrating and testing the proposed models, it was verified against results from other available models. Two comparisons were made. First, the average overall delays obtained from the simulation model for a given evaluation period under different saturation ratios were compared to the results from the Australian (Akcelik, 1981), Canadian (Teply et al., 1995), HCM (TRB, 1994) and Markov chain models (Olszewski, 1994). For convenience, the scenario used in this comparison is the same as that used by Olszewski (1994), who used it for a similar purpose. The evaluation period duration is 15 minutes. The signal timing consists of a cycle time of 60 seconds, an effective green interval of 24 seconds and a saturation flow of 1800 pcu/hr. A total of 6000 cycles, corresponding to 100 hours of traffic flow, was simulated for each degree of saturation. It was estimated that this number of simulations would result in an estimation error of less than 0.5 seconds at a significance level of 95%.

Figure 4 illustrates the average overall delay obtained from the simulation model and the four other methods. It should be noted that the overall delays associated with the HCM model have been obtained by multiplying the stopped delays from the HCM formula by 1.3 to convert stopped delay to overall delay. The Markov chain model assumes Poisson arrivals and constant departure during the green interval. As it would be expected, the

simulation results are almost identical to the Markov chain model. Among the three other models, the Australian model shows the best agreement with the simulation model under all levels of saturation and the Canadian model provides the best agreement with the simulation for oversaturated conditions. It should be noted that the differences between the HCM, Canadian and Australian delay equations are expected and have been addressed by Akcelik (1988).

The objective of the second comparison is to provide an indication of the validity of the simulation model in estimating the variance of delays. The simulation results are compared to those reported by Olszewski (1994) in which the exact means and variances of delays under various levels of saturation were obtained for a given case from a Markov chain model. The system parameters are the same as for the previous comparison except the evaluation time is 30 minutes, instead of 15 minutes. In this comparison, the number of cycles to be simulated was estimated on the basis of an analysis of the confidence interval for the variance. It was estimated that a total of 6000 cycles for each degree of saturation would yield an estimation error for the standard deviation of less than two seconds at a significance level of 95%. Figure 5 shows that the standard deviations of delay estimated by the simulation model and provided by Olszewski (1994) from the Markov chain model. It can be observed that the estimates of the standard deviation of delay from the simulation model are quite consistent with those obtained from the Markov chain model. The overestimation of the standard deviation of delay by the simulation model, especially in the range $x_t < 1.0$, is expected because the Markov chain model does not consider the variation of travel time within the cycle as quantified by Equation 8.

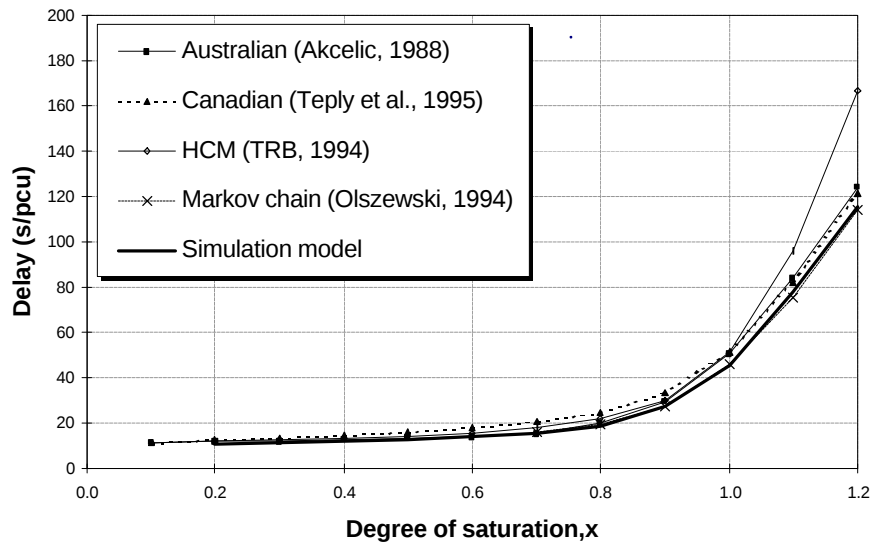


Figure 4. Average overall delay estimated by the Australian, Canadian, HCM, Markov chain and simulation models ($c_y = 60s$, $g_e = 24s$, $s = 1800$ pcu/h and $t = t_e = 15$ min; simulated cycles = 6 000)

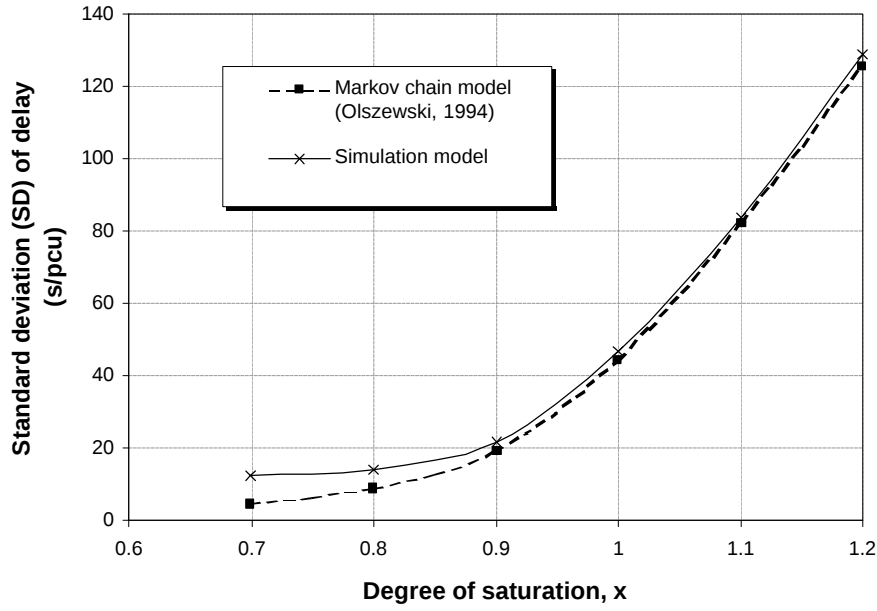


Figure 5 Standard deviation of delay estimated by the Markov chain model and simulation model ($c_y = 60s$, $g_e = 24s$, $s = 1800$ pcu/h and $t = t_e = 30$ min; simulated cycles = 6 000)

Calibration of Model for the Variance of Arrival Time Dependent Overflow Delay

To determine the appropriate parameter values for the arrival time dependent overflow delay variance model shown in Equation (15), a two-step sequential calibration procedure is performed. The first step is to find the x_0 and b values that would produce the best fit between the estimates from Equation (15) and the estimates from the simulation model (representing the true values) for a given cycle time (c), effective green interval (g_e) and prediction time (t). Calibration is accomplished by first transforming Equation (15) equivalently into an equivalent linear equation (Equation 17)

$$Y = a + b X \quad (17)$$

Where:

$$Y = \ln \left\{ \ln \left(\frac{tx_t}{c_a} \right) - \ln(\text{Var}[D_{t2}]) \right\}$$

$$X = \ln(x_t)$$

$$a = b \ln(x_0)$$

$$b = -b$$

The simulation model is used to obtain values of the variance of arrival time dependent overflow delay ($\text{Var}[D_{t2}]$) as a function of c_a , x_t and t . These data were transformed to X and Y values as in Equation 17. Linear regression was performed to determine the values of a and b , which were subsequently transformed back to values for x_0 and b . The data

points used in regression were determined by simulation by fixing the values of x_t , g_e and t and varying the degree of saturation from 0.7 to 1.2 with an increment of 0.05. Each data point is the result from a simulation of 15000 time intervals. The regressed x_0 and b , together with (c_y, g_e, t) , form a new data point (c_y, g_e, t, x_0, b) . By changing the values of the parameter set (c_y, g_e, t) and repeating the regression analysis, a number of such data points can be obtained. In this study, a total of 18 points were generated with the following combinations of parameters: $c_y = \{60, 120\}$; $l = \frac{g_e}{c_y} = \{0.2, 0.5, 0.8\}$; $t = \{300, 900, 1500\}$.

It was found that the linear relationship shown in Equation (17) is statistically significant for each of the 18 combinations with a minimum R^2 of 0.94.

In the second step, a series of correlation analysis of the relationships between the parameters (x_0, b) and $(c_y, g_e, t, l, t/c_d)$ were conducted and the following best fit equations were obtained:

$$x_0 = 0.928 + 0.069l \quad (18)$$

$$(df = 16, R^2 = 0.87, t_1 = 10.27)$$

$$b = 3.392 + 0.052t + 5.364l \quad (19)$$

$$(df = 15, R^2 = 0.93, t_1 = 13.07, t_2 = 3.84)$$

The obtained high R^2 values indicate that both equations explain a large portion of the variations in the simulated data. All t-values are greater than the critical t-value at the 5% level of significance, which indicates that the included parameters are statistically significant.

Model Evaluation

The simulation system is first used to estimate the mean and variance of real time delays corresponding to various arrival times and traffic conditions. A total of 42 combinations were simulated with the following combinations of parameters: $c_y = \{50, 100\}$; $l = \{0.2, 0.5, 0.8\}$; $t = \{300, 600, 900, 1200, 1500, 1800, 2100\}$. Figure 6 illustrates the correlation between the mean arrival time dependent delay estimated by the model (Equation 6) and those obtained from the simulation model. The approximate model exhibits no apparent bias and has a high correlation with the simulated estimates ($R^2=99.3\%$).

Figure 7 shows the correlation between the standard deviations of the arrival time dependent delay obtained by the analytical model with the simulation results. It is evident from Figure 5 that the model slightly underestimates the variance for undersaturated conditions ($x_t < 0.9$), but overall provides results remarkably consistent with the simulation with a R^2 of 99.1%. This is not unexpected because the model was calibrated with degrees of saturation ranging from 0.8 to 1.2.

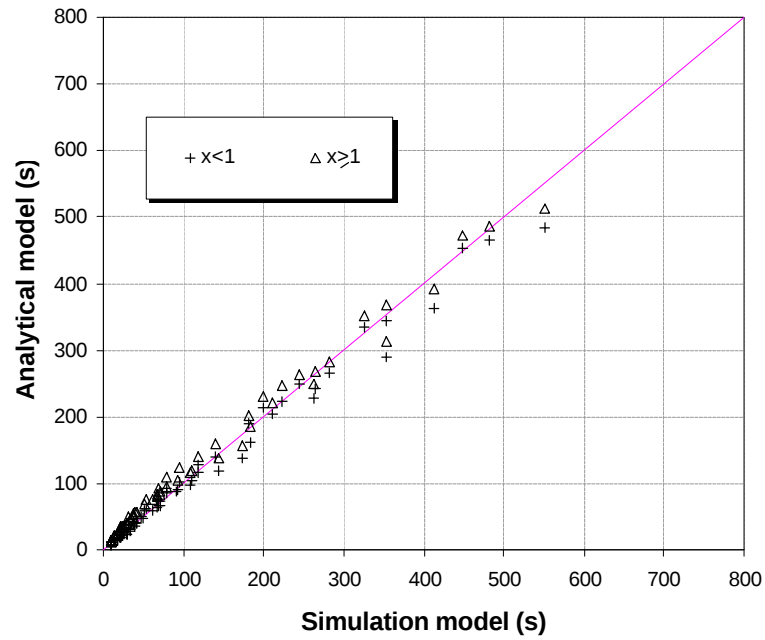


Figure 6. Correlation of mean arrival time dependent delay estimated by the analytical model with simulation results ($s = 1800$ pcu/h; simulated cycles = 15000)

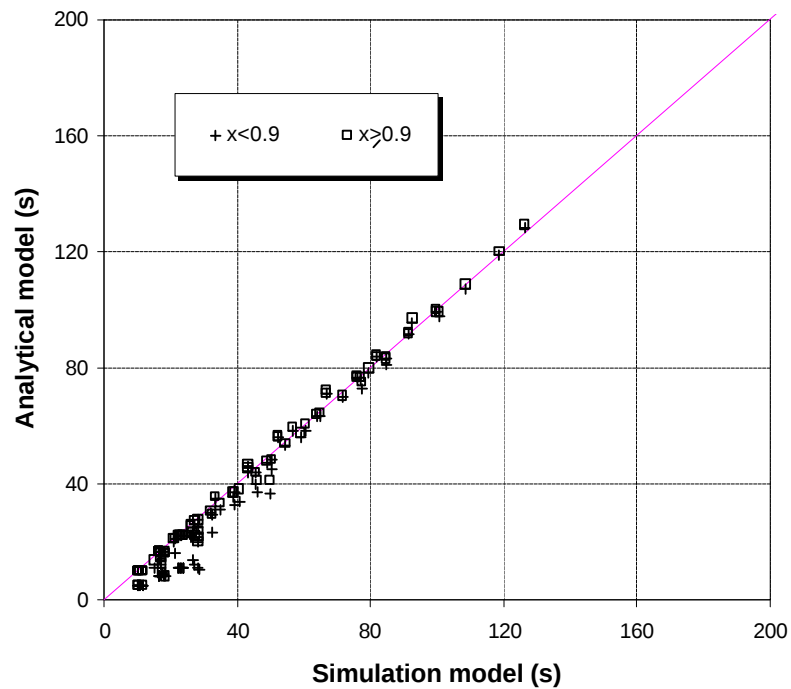


Figure 7. Correlation of the standard deviations of arrival time dependent delay estimated by the analytical model with simulation results ($s = 1800$ pcu/h; simulated cycles = 15000)

Delay Variability: A Sensitivity Analysis

The objective of this section is to demonstrate the application of the developed models by using them to analyze the effects of various factors on the arrival time dependent delay variability. Two measures were used to represent the delay variability. One is the standard deviation of arrival time dependent delay representing the absolute variability of delay and the other is the coefficient of variation (defined as the ratio of the standard deviation to the mean) of arrival time dependent delay representing the relative variability of delay. The influencing factors considered in this analysis were limited to the degree of saturation reflecting the level of traffic congestion (x_t) and the green to cycle ratio representing the traffic signal setting (l) and the vehicle arrival time (t). The analysis was performed on a case with a fixed cycle time of 100 seconds and a saturation flow of 1800 pcu/h. The point in time at which the arrival time dependent delays are to be estimated is 15 minutes after the time 0, that is, $t = 15$ min.

Figure 8 shows the standard deviations of delay as a function of the degree of saturation under the effective green-to-cycle ratios of 0.3 and 0.7. In general, the more congested the traffic is, the larger the variance of delay becomes. However, it is interesting to note that the standard deviation of delay is almost constant for the undersaturated traffic conditions ($x_t < 0.8$) with a value largely depending on the variance of uniform delay (Equation 8). This implies that the variance of the uniform arrival time dependent delay is insensitive to the level of traffic congestion. This also indicates that if only uniform delay is considered, as in Rouphail (1993), the variance would be significantly underestimated for saturated traffic conditions. From Figure 8, it can also be observed that the green proportion allocated to the approach has a important impact on the variance of delay.

Figure 9 illustrates the relationships between the coefficient of variation of delay and the degree of saturation under the effective green to cycle ratios of 0.3 and 0.7. This relationship is significantly different from the relationship between the absolute variability and the degree of saturation. Specifically, there is a higher degree of non-linearity between the relative variability of delay and the degree of saturation. The relative variability decreases in a linear fashion as the degree of saturation increases under light traffic conditions ($x_t < 0.8$). Under congested traffic conditions ($x_t > 0.8$), the relationship is highly non-linear. The relative variability increases as x_t approaches approximately 1.0 and then begin to decrease. It is possible to determine this point of inflection in the congested regime analytically from Equation 16.

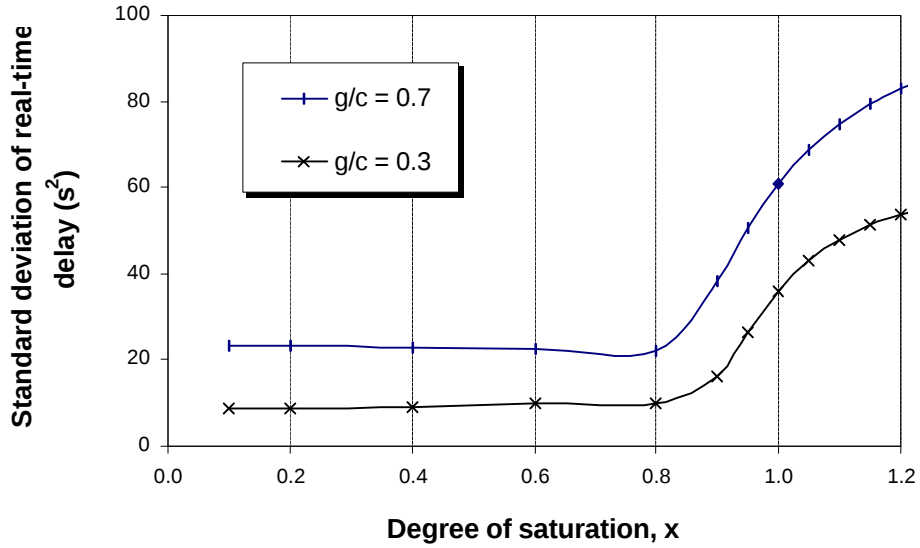


Figure 8. Relationship between the standard deviations of arrival time dependent delay and degree of saturation ($s = 1800$ pcu/h; $c_y = 100$ seconds; $t = 15$ min.)

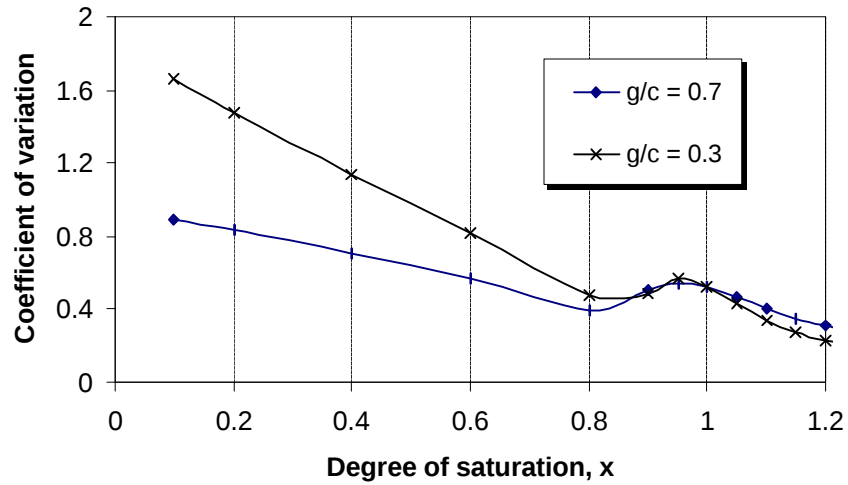


Figure 9. Relationship between the coefficient of variation of arrival time dependent delay and degree of saturation ($s = 1800$ pcu/h; $c_y = 100$ seconds; $t = 15$ min.)

CONCLUSIONS AND FUTURE RESEARCH

The ability to estimate link travel time as a function of future arrival time on the link is a key enabling technology for the successful deployment of many ITS systems, such as in-vehicle route-guidance systems and advanced traffic management systems. Current travel time estimation techniques, that are suitable for real-time applications, are generally limited to providing estimates of the means, based mainly on statistical data from probe vehicles and loop detectors. This paper has described the development of two approximate models for predicting the mean and variance of arrival time dependent delay at signal controlled approaches - the major component of the travel time on signalized arterial links. A discrete cycle-by-cycle simulation model was developed and used to

generate data for calibrating and validating the proposed models. The results of a correlation analysis indicates a remarkable agreement between the model estimates of the mean arrival time dependent delay ($R^2 = 99.3\%$) and the standard deviation of arrival time dependent delay ($R^2 = 99.1\%$), and simulation results.

The proposed analytical models were calibrated and validated with simulation results that are based on several important assumptions such as no existing initial queue, random traffic arrivals within a single traffic stream and unlimited queuing space. These assumptions, particularly the assumption of no initial queue, may be overly restrictive and is likely to be violated in practice. The impact of these assumptions on the validity of these models has not yet been determined. It is recommended that future research focus on the following aspects.

- First, the potential impacts of the assumptions applied in this paper should be quantified.
- Second, the potential benefits of using these models within in-vehicle route guidance systems and traffic management systems should be evaluated;
- Field data should be used in conjunction with simulation results to calibrate and verify the proposed models;
- A more complete sensitivity analysis should be performed on the effects of all relevant factors on the variability of the arrival time dependent delay.

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REFERENCES:

- Abours, S. (1986) Estimation of Travel Time from Occupancy on an Urban Network, an Experiment in Paris, France. Second International Conference on Road Traffic Control, Institution of Engineers, London, UK, 1986.
- Akcelik, R. (1981) Traffic Signals: Capacity and Timing Analysis, Research Report No. 123, Australian Road and Research Board, Vermont South.
- Akcelik, R. (1988) "The Highway Capacity Manual Delay Formula for Signalized Intersections" ITE Journal 58 (3), pp. 23-27.
- Allsop, R. E. (1972) "Delay at a Fixed Time Traffic Signal – I: Theoretical Analysis. Transportation Science, Vol. 6, 1972, pp. 260-285.
- Boyce, D., Roupail, N., and A. Kirson, (1993), "Estimation and Measurement of Link Travel Times in the ADVANCE Project", IEEE-IEE Vehicle Navigation & Information Systems Conference, Ottawa - VINS'93.
- Brilon, W. And Wu, N. (1990) Delays at Fixed Time Traffic Signals Under Time-Dependent Traffic Condition, Traffic Engineering and Control 31 (12), pp. 623-631.)

- Cronje, W.B. (1983), "Derivation of Equations for Queue Length, Stops, and Delay for Fixed-Time Traffic Signals" *Transportation Research Record* 905, TRB, Washington D. C., 1983.
- Fu, L. and S. Teply, "Improving the Reliability of Paratransit Service Using Better Travel Time Information", Proceeding of the Joint Conference of the Canadian Institute of Transportation Engineers, Pacific Northwest ITE Quad Section and Western Canada Traffic Association, April 13~16, 1997, Vancouver, British Columbia
- Fu, L. Real-Time Vehicle Routing and Scheduling in Dynamic and Stochastic Traffic Networks, Ph. D. Dissertation, University of Alberta, Edmonton, Alberta, 1996.
- Hock, N. C., *Queueing Modelling Fundamentals*, John Wiley & Sons, Baffins Lane, Chichester, England, 1996
- Kimber, R. M. and Hollis, E. M. (1979) Traffic Queues and Delays at Road Junctions. LR 909, Transportation Road Research Laboratory, Crowthorne.
- May, D. A. *Traffic Flow Fundamentals*, Prentice Hall, Englewood Cliffs, New Jersey, 1990.
- Medhi, J. Stochastic Models in Queueing Theory, Academic Press, Inc. Harcourt Brace Jovanovich Publishers, San Diego, 1991
- Newell, G. F. 1982, *Applications of Queueing Theory*, Chapman Hall, London.
- Olszewski, P. (1993) "Modeling of Queue Probability Distribution at Traffic Signals. In *Transportation and Traffic Theory*, ed. M. Koshi, Elsevier, New York.
- Olszewski, P. (1994) "Modeling Probability Distribution of Delay at Signalized Intersections. *Journal of Advanced Transportation*, Vol. 28, No. 3, pp. 253-274.
- Roughail, M. N. and N. Dutt, (1995), "Estimating Travel Time Distribution for Signalized Links: Model Development and Potential IVHS Applications", Proceeding of the annual meeting of ITS America, Vol. 1, Washington, D. C., March 15-17, 1995.
- Roughail, N. M. and Akcelik, R. (1990) "Oversaturation Delay Estimates with Consideration of Peaking, Transportation Research Record 1365, TRB, Washington D. C.. pp. 71-81.
- Sen, A., P. Thakuriah, X. Zhu and A. Karr, (1997) "Frequency of Probe Reports and Variance of Travel Time Estimates", *Journal of Transportation Engineering*, Vol. 123, No. 4, 1997.
- Sen, A., S. Soot, J. Ligas and X. Tian, (1997) "Arterial Link Travel Time Estimation: Probes, Detectors and Assignment-type Models. Preprint No. 970943, Transportation Research Board, 76th Annual Meeting, Washington, D. C.
- Takaba, S. "Estimation of Measurement of Travel Time by Vehicle Detectors and License Plate Readers, Second Vehicle Navigation and Information Systems Conference. Michigan, Society of Automotive Engineers, Warrendale, PA., 1991.

- Teply, S. and G. D. Evans, (1989), "Evaluation of the Quality of Signal Progression by Delay Distributions." *Transportation Research Record* 1225, TRB, Washington D. C., 1989.
- Teply, S., D. I. Allingham, D. B. Richardson and B. W. Stephenson, *Canadian Capacity Guide for Signalized Intersections*, Second Edition (S. Teply, ed.), Institute of Transportation Engineering, District 7, Canada, 1984.
- Teply, S., D. I. Allingham, D. B. Richardson and B. W. Stephenson, *Canadian Capacity Guide for Signalized Intersections*, Second Edition (S. Teply, ed.), Institute of Transportation Engineering, District 7, Canada, 1995.
- TRB, (1994), *Highway Capacity Manual*, 1994 Update, Special Report 209, Transportation Research Board, National Research Council, Washington, D. C., 1985.
- Van Aerde, M., B. Hellinga, L. Yu, and H. Rakha (1993) Vehicle Probes as Real-Time ATMS Sources of Dynamic O-D Travel Time Data. Large Urban Systems - *Proceedings of the ATMS Conference*, in St. Petersburg, Florida. pp. 207- 230.